**QUESTION ONE**

1. **pΛ~q**

|  |  |  |  |
| --- | --- | --- | --- |
| **p** | **q** | **~q** | **pΛ~q** |
| **T** | **T** | F | F |
| **T** | **F** | T | T |
| **F** | **T** | F | F |
| **F** | **F** | T | F |

1. **~pΛ~q**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **p** | **q** | **~p** | **~q** | **~pΛ~q** |
| **T** | **T** | F | F | F |
| **T** | **F** | F | T | F |
| **F** | **T** | T | F | F |
| **F** | **F** | T | T | T |

1. **~(~pΛq)**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **p** | **q** | **~p** | **~pΛq** | **~(~pΛq)** |
| **T** | **T** | F | F | T |
| **T** | **F** | F | F | T |
| **F** | **T** | T | T | F |
| **F** | **F** | T | F | T |

1. **~(~pv~q)**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **p** | **q** | **~p** | **~q** | **~p v ~q** | **~(~p v ~q)** |
| **T** | **T** | F | F | F | T |
| **T** | **F** | F | T | T | F |
| **F** | **T** | T | F | T | F |
| **F** | **F** | T | T | T | F |

1. **∼(∼P V Q)**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **P** | **Q** | **~P** | **(~P V Q)** | **~(~P V Q)** |
| **T** | **T** | F | T | F |
| **T** | **F** | F | F | T |
| **F** | **T** | T | T | F |
| **F** | **F** | T | T | F |

1. **P V (∼P Λ Q)**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **P** | **Q** | **~P** | **~P Λ Q** | **P V (∼P Λ Q)** |
| **T** | **T** | F | F | T |
| **T** | **F** | F | F | T |
| **F** | **T** | T | T | T |
| **F** | **F** | T | F | F |

**QUESTION TWO**

1. ***Set A = {1,0,3} ; Set B = {4,5,6}; Set C = {7,8,9}. Proof properties of sets (set laws) using the sets above***
2. **Commutative law:**

A U B = {1,2,3,4,5,6} & B U A = {4,5,6,1,2,3}

Elements in A U B and elements in B U A are the same, hence, **A U B = B U A**

A n B ={ } and B n A = { }

There are no elements in A n B, just as there are no elements in B n A, hence, **A n B = B n A**

1. **Absorption Law**

(A u B) n A = A

A u B = {1,2,3,4,5,6}

(A u B) n A = {1,2,3}

Therefore, **( A u B) n A = A**

(A n B) u A = A

(A n B) = { }

(A n B) u A = {1,2,3}

Therefore **(A n B) u A = A**

1. **De Morgan’s Law**

A = (B u C) = (A – B) n (A – C)

(B u C) = {4,5,6,7,8,9}

A – (B u C) = { 1, 2, 3}

A-B = {1,2,3} A-C = {1,2,3}

A = (B n C) = (A – B) u (A – C)

Therefore, **A – (B u C) = (A – B) n (A – C)**

A = (B n C) = (A – B) u (A – C)

B n C = { }

A – (B n C) = {1,2,3}

A – B = {1,2,3}

A – C ={1,2,3}

(A – B) u (A – C) = {1,2,3}

Therefore, **A-(B n C) = (A – B) u (A – C)**

1. **Distributive law**

A n (B u C):

B u C = {4,5,6,7,8}

A n (B u C) = {}

(A n B) u (A n C)

A n B = { } & A n C = { }

(A n B) u (A n C) = { }

Therefore, **A n (B u C) = (A n B) u (A n C)**

A u (B n C) = (A u B) n (A u C)

A u (B n C)

B n C = { }

A u (B n C} = {1,2,3}

A u B = {1,2,3,4,5,6}

A u C = {1,2,3,7,8,9}

(A u B) n (A u C)= {1,2,3}

Therefore, **A u (B n C) = (A u B) n (A u C)**

1. **Independent law**

A u A = {1,2,3} & A = {1,2,3}

Therefore, **A u A = A**

A n A = {1,2,3} & A = {1,2,3}

Therefore **A n A = A**

1. **Associative law**

(A u B) u C = A u (B u C)

A u B = {1,2,3,4,5,6}

(A u B) u C = {1,2,3,4,5,6,7,8,9}

(B u C) = {4,5,6,7,8,9}

A u (B u C) = {1,2,3,4,5,6,7,8,9}

Therefore **(A u B) u C = A u (B u C)**

(A n B) n C = A n (B n C)

A n B = { }

(A n B) n C= { }

B n C = { }

A n (B n C) = { }

Therefore **(A n B) n C = A n (B n C)**

**QUESTION THREE**

1. **Proove that ~p v p is a tautology,**
2. **Substituting qΛr for p, we obtain proposition (qΛr) V~ (qΛr). Verify that it is a tautology too**

|  |  |  |
| --- | --- | --- |
| **p** | **~p** | **~p v p** |
| **T** | **F** | **T** |
| **F** | **T** | **T** |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **q** | **r** | **q Λ r** | **~ (q Λ r)** | **(q Λ r) V ~ (q Λ r)** |
| **T** | **T** | T | F | **T** |
| **T** | **F** | F | T | **T** |
| **F** | **T** | F | T | **T** |
| **F** | **F** | F | T | **T** |

**QUESTION FOUR: Proove that this is a tautology [(p→q) Λ(q→r)]→(p→r)**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **p** | **q** | **r** | **p→q** | **q→r** | **(p→q) Λ(q→r)** | **[(p→q) Λ(q→r)]→(p→r)** |
| **T** | **T** | **T** | T | T | T | T |
| **T** | **T** | **F** | T | F | F | T |
| **T** | **F** | **T** | F | T | F | T |
| **T** | **F** | **F** | F | T | F | T |
| **F** | **T** | **T** | T | T | T | T |
| **F** | **T** | **F** | T | F | F | T |
| **F** | **F** | **T** | T | T | T | T |
| **F** | **F** | **F** | T | T | T | T |

**QUESTION FIVE**

1. **qΛ(p→q)→p**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **p** | **q** | **p→q** | **qΛ (p→q)** | **qΛ (p→q) →p** |
| **T** | **T** | T | T | T |
| **T** | **F** | F | F | T |
| **F** | **T** | T | T | F |
| **F** | **F** | T | F | T |

1. **~(pΛq)↔(~pV~q)**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **p** | **q** | **pΛq** | **~(pΛq)** | **~p** | **~q** | **(~pV~q)** | **~(pΛq)↔(~pV~q)** |
| **T** | **T** | T | F | F | F | F | T |
| **T** | **F** | F | T | F | T | T | T |
| **F** | **T** | F | T | T | F | T | T |
| **F** | **F** | F | T | T | T | T | T |

**QUESTION SIX**

***Possible subsets of ∑\* include***: { } , {a}, {b}, {ab}, {ba}